PROOF:

a) Assume that f and g are injective. Take $a,b \in X$. $(g \circ f)(a) = (g \circ f)(b) \Rightarrow g(f(a)) = g(f(b))$. Since g is injective, then $f(a) = f(b)$ and since f is injective, then $a = b$. Hence $g \circ f$ is injective.

b) Assume that f and g are surjective. Take $z \in Z$. Since g is surjective, then $\exists y \in Y : g(y) = z$ and since f is surjective, then $\exists x \in X : f(x) = y$. Hence $z = g(y) = g(f(x)) = (g \circ f)(x)$ i.e., $g \circ f$ is surjective.

c) Assume that $g \circ f$ is injective. Take $a,b \in X$. $f(a) = f(b) \Rightarrow g(f(a)) = g(f(b)) \Rightarrow (g \circ f)(a) = (g \circ f)(b) \Rightarrow a = b$. Hence f is injective.

d) Assume that $g \circ f$ is surjective. Take $z \in Z$. Since $g \circ f$ is surjective, then $\exists x \in X : (g \circ f)(x) = z \Rightarrow \exists x \in X : g(f(x)) = z$. If we define $y := f(x) \in Y$, then we obtain $\forall z \in Z,\ \exists y \in Y :\ g(y) = z$. Hence g is surjective.