PROOF:

i) R is reflexive because $\forall a \in X$, $\exists i \in I$: $a \in A_i \Rightarrow aRa$.

ii) R is symmetric because $aRb \Rightarrow \exists i \in I$: $a,b \in A_i \Rightarrow b,a \in A_i \Rightarrow bRa$.

iii) $aRb \wedge bRc \Rightarrow \exists i, j \in I$: $a,b \in A_i \wedge b,c \in A_j$. Then, $b \in A_i \cap A_j \neq \varnothing$. Since the family $\{A_i\}_{i \in I}$ is a partition of X , it is pairwise disjoint. Then, it is obtained $A_i = A_j$ and $i = j$. Consequently, it holds aRc from $a,c \in A_i$.

It has been proved R is an equivalence relation. Now, let's prove that the equivalence classes of R is the family $\{A_i\}_{i \in I}$:

Firstly, for $i \in I$, we will show that the set A_i is an equivalence class of R . We know from the hypothesis that $A_i \neq \varnothing$. Then, $\exists a_i \in X$: $a_i \in A_i$.

Since $x \in A_i \Leftrightarrow a_i R x \Leftrightarrow x \in \overline{a_i}$, then A_i is an equivalence class of R .

Secondly, assume $\overline{a}$ is an equivalence class of R . Then, $\exists i \in I$: $a \in A_i$. Since $x \in A_i \Leftrightarrow aRx \Leftrightarrow x \in \overline{a}$, then $\overline{a} = A_i$.