PROOF: We must show the followings to prove that this family is a partition of X :

1. $X=\bigcup_{a\in X}\overline{a}$
2. $\overline{a}=\overline{b}$ or $\overline{a}\cap\overline{b}=\varnothing$ for every $a,b\in X$.

Proof of (1): Since $\overline{a}\subset X$ for every $a\in X$, then $\bigcup_{a\in X}\overline{a}\subset X$. Besides, $\forall x\in X$,

$x\in\overline{x}\Rightarrow x\in\bigcup_{a\in X}\overline{a}\Rightarrow X\subset\bigcup_{a\in X}\overline{a}$.

Consequently, $X=\bigcup_{a\in X}\overline{a}$.

Proof of (2): It is enough to show $\overline{a}\cap\overline{b}\neq\varnothing\Rightarrow\overline{a}=\overline{b}$.

Assume $\overline{a}\cap\overline{b}\neq\varnothing$. Then, $\exists x\in X$: $x\in\overline{a}\wedge x\in\overline{b}\Rightarrow x\sim a\wedge x\sim b$ $\Rightarrow a\sim x\wedge x\sim b\Rightarrow a\sim b$.

Hence, it holds $\overline{a}=\overline{b}$ from Proposition1.

Consequently, the family $\left\{\overline{a}:a\in X\right\}\subset\mathrm{P}\left(X\right)$ is a partition of X .