PROOF:

i) Because the universal set includes any set, $\varnothing^C \subset E$ is provided. Let's prove its inverse: Let x is in E. It is obvious that $x \notin \varnothing$. Then $x \in \varnothing^C \Rightarrow E \subset \varnothing^C \Rightarrow \varnothing^C = E$.

ii) The proof of this proposition is rather obvious. Let's assume that $x \in E^C$. Then $x \notin E$. This is obviously a contradiction. So, the assumption $x \in E^C$ is false. I.e., E has no element. $E^C = \varnothing$.

iii) $A = \varnothing \Rightarrow \left(A^C\right)^C = \left(\varnothing^C\right)^C = E^C = \varnothing = A$.

$A = E \Rightarrow \left(A^C\right)^C = \left(E^C\right)^C = \varnothing^C = E = A$. Now, we consider $A \neq \varnothing$ and $A \neq E$. Because $x \in \left(A^C\right)^C \Leftrightarrow x \notin A^C \Leftrightarrow x \in A$, $\left(A^C\right)^C = A$ is obtained.

iv) Let $\left(\bigcup_{i\in I} A_i\right)^C = \varnothing$. Let's show that $\bigcap_{i\in I} A_i^C = \varnothing$. We assume the contrary i.e., $\bigcap_{i\in I} A_i^C \neq \varnothing$. Then $\exists x \in E:\ x \in \bigcap_{i\in I} A_i^C \ \Rightarrow\ \exists x \in E:\ \forall i \in I,\ x \in A_i^C \ \Rightarrow\ \exists x \in E:\ \forall i \in I,\ x \notin A_i$. I.e., the element x is not in any A_i $\left(i \in I\right)$. Consequently, the element x is not in $\bigcup_{i\in I} A_i$ (if the element x were in $\bigcup_{i\in I} A_i$, it had to be in at least one of the sets $\left\{A_i\right\}_{i\in I}$) i.e., $x \notin \bigcup_{i\in I} A_i \Rightarrow x \in \left(\bigcup_{i\in I} A_i\right)^C$. This contradicts the hypothesis. Our assumption is not true i.e., $\bigcap_{i\in I} A_i^C = \varnothing$.

Now, let's show the assertion in the case $\left(\bigcup_{i\in I} A_i\right)^C \neq \varnothing$.

$$x \in \left(\bigcup_{i\in I} A_i\right)^C \Leftrightarrow x \notin \bigcup_{i\in I} A_i \Leftrightarrow \sim [\exists i \in I : x \in A_i] \Leftrightarrow \forall i \in I, x \notin A_i \Leftrightarrow \forall i \in I, x \in A_i^C \Leftrightarrow x \in \bigcap_{i\in I} A_i^C .$$

Consequently,

$$\left(\bigcup_{i\in I} A_i\right)^C = \bigcap_{i\in I} A_i^C .$$

v) We consider the set A_i is replaced by A_i^C for all $i \in I$ in the proposition (iv). We have $\left(\bigcup_{i\in I} A_i^C \right)^C = \bigcap_{i\in I} \left(A_i^C \right)^C \Rightarrow \left(\bigcup_{i\in I} A_i^C \right)^C = \bigcap_{i\in I} A_i$ by using (iii). Taking complement of both sides of the last equation, we obtain $\left(\bigcap_{i\in I} A_i \right)^C = \bigcup_{i\in I} A_i^C$.