PROOF:

"$\Rightarrow$": Assume that f is injective. Since an injection maps n different elements in the domain to n different elements in the codomain, then $\operatorname{Im} f$ has n different elements. Further, since Y has n elements, then $\operatorname{Im} f = Y$. Consequently, f is surjective.

"$\Leftarrow$": Assume that f is surjective but not injective. Then there exist at least two elements in X such as the images of these are equal to each other. So, the number of the images of the elements in X is at most $n-1$ because X has strictly n elements. However, $Y = \operatorname{Im} f$ and Y has strictly n elements. This is a contradiction. Consequently, if f is surjective, then it is injective.