PROOF: It is clear that

$$a \sim b \Leftrightarrow a \in \overline{b} \Leftrightarrow b \in \overline{a}$$

from symmetry and definition of equivalence class. Hence, it is enough to prove that $a \sim b \Leftrightarrow \overline{a} = \overline{b}$.

Assume $a \sim b$.

$x \in \overline{a} \Rightarrow x \sim a$. Since $a \sim b$ and transitivity, $x \sim b \Rightarrow x \in \overline{b} \Rightarrow \overline{a} \subset \overline{b}$.

Similarly, $\overline{b} \subset \overline{a}$.

Assume $\overline{a} = \overline{b}$. Then, $a \in \overline{a} \Rightarrow a \in \overline{b} \Rightarrow a \sim b$